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VERIFIABLE DELAY FUNCTIONS

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VERIFIABLE DELAY FUNCTIONS

How to slow things down

VERIFIABLE DELAY FUNCTIONS

[Boneh, Bonneau, Bünz, Fisch 2018] A VDF is a function that

- Requires **time** to evaluate (sequential evaluation, and parallelism does not allow to go faster)
- The output can easily be verified

Syntactically:

➡ **setup**$(T) \rightarrow$ public parameters pp

➡ **eval**$(pp, x) \rightarrow$ output y, proof π (takes time T)

➡ **verify**$(pp, x, y, \pi) \rightarrow$ {true, false}

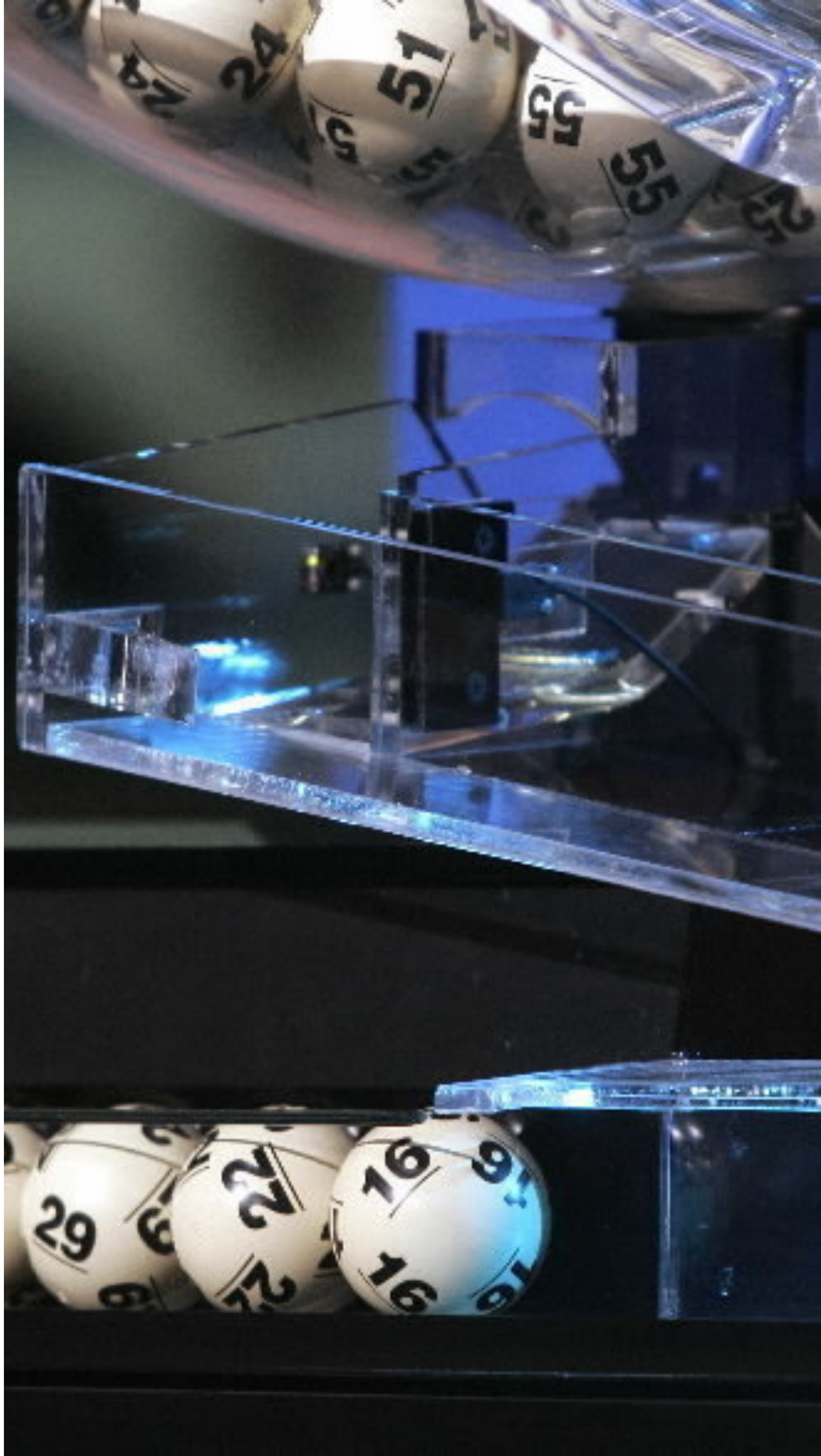
REQUIREMENTS

We need the following properties:

- **Sequentiality:** if A is a parallel algorithm such that time(A, x) $< T$, then A cannot distinguish **eval**(pp, x) from random
- **Uniqueness:** if **verify**(pp, x, y, π) $=$ **verify**(pp, x, y', π') $=$ true, then $y = y'$

PUBLIC RANDOMNESS

A motivation



AD HOC "METHODS"

A CRYPTOGRAPHIC ATTEMPT

A group G of people want to generate some randomness:

- Each person $A \in G$ generates privately a random bit-string r_A
- They all broadcast a commitment $c(r_A)$ (hiding, binding)
- Once all the commitments are distributed, everyone opens
- Random value is $r = \bigoplus_{A \in G} r_A$

'Commit-then-reveal' protocol

A CRYPTOGRAPHIC ATTEMPT

- Two rounds
- Does not scale!
- If someone does not open the commitment, need to restart

'Commit-then-reveal' protocol

SLOTH AND UNICORN

Solution proposed in [Lenstra, W. 2017]:

- Instead of commitments, each party A directly reveals r_A

No commitment, so no 'opening' phase

Trouble: last person to reveal has full control of $r = \bigoplus_{A \in G} r_A$...

- Instead, let $r = f(r_{A_1} \,||\, r_{A_2} \,||\, \ldots \,||\, r_{A_n})$, where f takes **time** to evaluate (in [Lenstra, W. 2017] the *Sloth* function)

If f takes 10 minutes, nobody knows r
until 10 minutes after the last reveal:
impossible to manipulate r!

VERIFIABLE DELAY FUNCTION

We want

- $f(x)$ **slow** to evaluate, even for parties with a lot of parallel power or specialised hardware
- $f(x) = y$ easy to **verify** by anyone

Use a **verifiable delay function**

A VERIFIABLE DELAY FUNCTION

Slow yet efficient

ITERATED HASHING

What is slow to compute, and cannot be sped up by parallelism? Maybe iterated hashing...

$$x \longrightarrow H(x) \longrightarrow H(H(x)) \longrightarrow \ldots \longrightarrow H(\ldots H(H(x))\ldots) = y$$

- Slow, sequential computation... but how to check $f(x) = y$?
- No simple and efficient way...

TIME LOCK PUZZLE

Drawing inspiration from time-lock puzzles [Rivest, Shamir, Wagner 1996]

- Let G be a group of unknown order
- Given $x \in G$, computing x^{2^T} requires T sequential squarings

$$x \longrightarrow x^2 \longrightarrow x^{2^2} \longrightarrow x^{2^3} \longrightarrow \ldots \longrightarrow x^{2^T}$$

- The VDF could be $f(x) = x^{2^T}$, but can this be verified?

Approach of [W. 2019], also taken in [Pietrzak 2019]

PROOF OF CORRECT EXPONENTIATION

- Given $(x, y) \in G$, Alice wants to prove that $y = x^{2^T}$
  - ➡ Together with $y = x^{2^T}$, Alice computes a 'proof' π
  - ➡ Given (x, y, π), anyone can efficiently verify that $y = x^{2^T}$
- We present the method as an interactive protocol: Alice wants to prove **to Bob (the verifier)** that $y = x^{2^T}$
- The protocols is then be made non-interactive (Fiat-Shamir...)

INTERACTIVE ARGUMENT

- Given $(x, y) \in G$, Alice wants to prove to Bob that $y = x^{2^T}$

Alice		Bob
	$\xleftarrow{\ell}$	Choose a random (large) prime ℓ
Find q and r such that $2^T = q\ell + r,\ 0 \leq r < \ell$	$\xrightarrow{\pi = x^q}$	
		Compute $r = 2^T \bmod \ell$
		Accept if $\pi^\ell x^r = y$

NON-INTERACTIVE VDF

The VDF on input $x \in G$ is the following:

- ➡ Compute $y = x^{2^T}$ (slow, sequential part)
- ➡ Let ℓ = hash_to_prime(x, y, T)
- ➡ Find q and r such that $2^T = q\ell + r$, and $0 \leq r < \ell$
- ➡ Compute $\pi = x^q$ How long does the computation of π take?
- ➡ Output: (y, π), only 2 group elements

- **verify**(pp, x, y, π): $\pi^\ell x^r = y$, only 2 small exponentiations

PROPERTIES

	Size of proof	Evaluation	Verifier
	number of group elements	number of group operations	
Sloth [Lenstra, W. 2017]	1	T	$O(T)$
[Pietrzak 2019]	$\log(T)$	$T(1 + 2/T^{1/2})$	$O(\log(T))$
This work [W. 2019]	1	$T(1 + 2/\log(T))$	$O(1)$

SECURITY

- Given $(x, y) \in G$, Alice wants to prove to Bob that $y = x^{2^T}$

Alice

Bob

Bob: Choose a random (large) prime ℓ

Bob → Alice: ℓ

Alice: Find q and r such that $2^T = q\ell + r,\ 0 \leq r < \ell$

Alice → Bob: $\pi = x^q$

Bob: Compute $r = 2^T \bmod \ell$

Accept if $\pi^\ell x^r = y$

SECURITY

- Suppose $y \neq x^{2^T}$ (i.e., Alice is dishonest)
- Let $w = y/x^{2^T} \neq 1_G$
- **Claim:** for Alice to convince Bob, she must be able to extract ℓ-th roots of w with good probability (unpredictable ℓ)
- **Proof:** when Bob generates a random ℓ, Alice computes π such that $\pi^\ell x^r = y$ (acceptance condition), where $2^T = q\ell + r$. Let $\rho = \pi/x^q$. Then,

$$\rho^\ell = \pi^\ell/x^{q\ell} = (y/x^r)/x^{q\ell} = y/x^{q\ell + r} = w$$

i.e., ρ is an ℓ-th root of w

ADAPTIVE ROOT ASSUMPTION

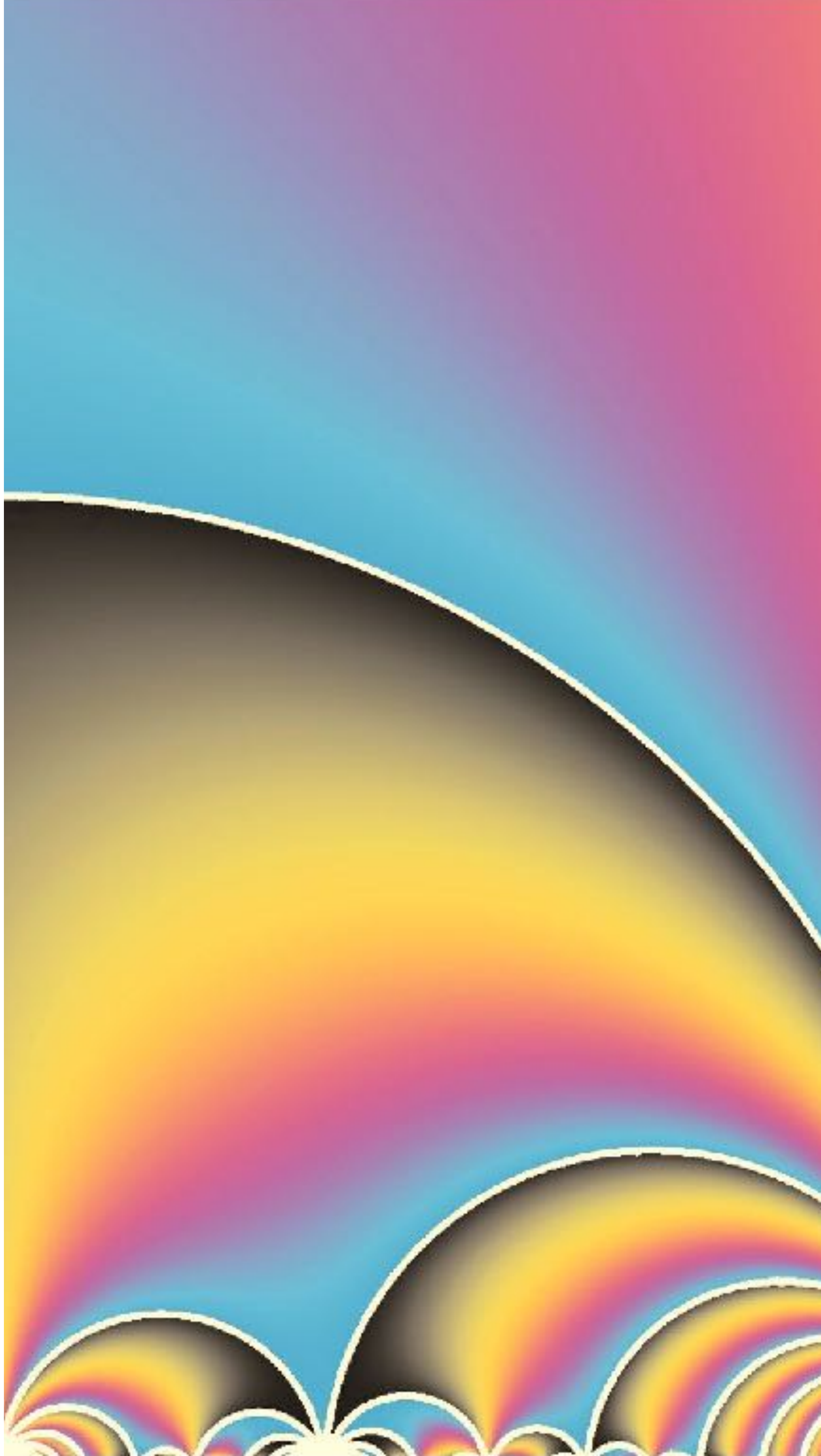
We assume the following game is hard in the group G:

- The player outputs an element $w \in G$, other than the neutral element 1_G
- The challenger generates a random (large) prime ℓ
- The player has to find an ℓ-th root of w (i.e., $w^{1/\ell}$)

In which groups does this assumption hold?

GROUPS OF UNKNOWN ORDER

From number theory



THE PROBLEM WITH KNOWN ORDER

- Suppose $w \in G$ has known order n
- The challenger generates a random (large) prime ℓ
- Computing $k = \ell^{-1} \bmod n$ is easy (invertible with overwhelming probability)
- w^k is an ℓ-th root of w

RSA GROUPS

Let $N = pq$ an RSA modulus

- Without the factorisation of N, order of $(\mathbb{Z}/N\mathbb{Z})^\times$ is unknown
- We still know the small subgroup $\{\pm 1\}$... trouble
- Use $G = (\mathbb{Z}/N\mathbb{Z})^\times/\{\pm 1\}$
- **Problem:** need to generate N so that nobody knows the factorisation (trusted setup? large random N? MPC?)

RSA MPC

Goal of the Ethereum Foundation and Protocol labs, working with Ligero:

- A 2048 bits modulus N, secret factorisation
- Result of an $(n-1)$-maliciously secure MPC
- 1024 participants

CLASS GROUPS

Let p be a random large prime, K the imaginary quadratic field of discriminant $-p$, and G its class group

- Computing the order of G is hard (complexity $L_p(1/2)$)
- Easy setup! Can even change p at every new evaluation... becomes 'quantum resistant'
- Careful: the 2-torsion is easy to compute

ADAPTIVE ROOT ASSUMPTION

- Open question:« adaptive root assumption » is not known to be equivalent to finding an element of known order
- It is hard in the generic group model [Boneh, Bünz, Fisch 2018]
- Is it as hard as it looks in RSA groups and class groups? At least, root extraction (non-adaptive) is believed to be hard

SLOWNESS IN THE REAL WORLD

Practical considerations

TIME LOCK ASSUMPTION

Assumption: computing

$$x \longrightarrow x^2 \longrightarrow x^{2^2} \longrightarrow x^{2^3} \longrightarrow \dots \longrightarrow x^{2^T}$$

takes time $\approx T \times$ (latency of one squaring in the group)

- What is that latency?
- Can a rich adversary get a much better latency than easily available hardware?

Solution: massively invest in building the fastest hardware, and make it widely available

$100,000 COMPETITION

Chia Network organises a VDF competition (second round finished Jul 18 with $100,000 in total prize money)

- Fastest possible implementation of **class group arithmetic**
- https://www.chia.net

$1,000,000 COMPETITION

Funded 50/50 by the Ethereum Foundation and Protocol Labs

- Fastest possible implementation of **modular arithmetic**, modulo a 2048-bit RSA modulus
- Latency of 1ns per squaring?
- https://vdfresearch.org

LOWER BOUNDS?

Let

$$\text{MODSQ-MOD2}_{b,N} : \{0, 1\}^b \longrightarrow \{0, 1\}$$

the function that sends x to the least significant bit of (x^2 mod N)

Theorem [W., Williams 2020]: For all odd $0 \leq N \leq 2^b - 1$, every fan-in two circuit of depth less than $\log_2(b - O(1))$ fails to compute $\text{MODSQ-MOD2}_{b,N}$ on at least 24% of all b-bit inputs

In simpler words: A circuit that performs « squaring modulo N » in binary representation reliably has depth at least $\approx \log_2(b)$

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